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ABSTRACT

This paper is addressed to the importance of maintaining rigor (a strict adherence to certain principles of reasoning) in the teaching of mechanics. The importance of a strict interpretation of mathematical formulae and the necessity of rigorous definitions of time, mass, and force are shown through a series of examples. Implications of this idea are elaborated in sections dealing with the teachings of dynamics and kinematics. (CP)

THE ROLE OF RIGOR IN THE TEACHING OF MECHANICS

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RIGOR AND UNDERSTANDING. In the preface to his book, General Mechanics, Max Planck* said: "I have frequently observed that the difficulties with which the student has to contend when he first enters the realm of theoretical physics are more often concerned, not with the mathematical form, but with the physical content of the ideas that are presented to him. It is not the calculations with equations that cause him the most trouble, but the setting up of the equations, and, in particular, their interpretation." Knowledge of engineering mechanics implies more than the ability to derive and interpret the basic equations of mechanics. It signifies the ability to apply the principles of mechanics to various physical systems, and thereby to predict the mechanical behavior of the systems correctly. Experience plays an important part in the development of such understanding, but the first requisite is an insight into the principles of mechanics. Planck's imaginative and deductive treatment of classical physics, illuminated by an easy style, displays mathematics, not as a tool, but rather as the language of physics. The role of rigor in mechanics is partly the cultivation of understanding and fluency in this language.

According to the dictionary, rigor means strictness or severity. Rigor, in this sense, whether self-imposed or enforced by authorities, is essential for learning. Students like a teacher, who is a good fellow, but they are apt to learn more from one who insists on complete explanations, clear reasoning, neatness, and accuracy.

However, in the present context, rigor is not viewed from the standpoint of the disciplinarian, important though that may be. In mathematics, rigor means strict adherence to certain principles of reasoning. It does not mean ultimate logic, for that may be unknowable; at least, it still eludes philosophers and mathematicians. Despite misgivings aroused by Goedel's famous demonstration that a postulational approach to a science may enmesh us in inconsistencies and deny us access to certain truths, rigor in mathematics generally is interpreted to imply the axiomatic method. D. Hilbert said: "I think that everything that can be an object of scientific study at all, as soon as it is ripe for the formation of a theory, falls into the lap of the axiomatic method and thereby indirectly of mathematics. Under the banner of the axiomatic method, mathematics seems destined for a leading role in science." The axiomatic method originated in classical Greece, but it was not understood clearly until this century.

*General Mechanics, MacMillan Co., New York, 1932.

Following the pattern of Euclid, Newton attempted an axiomatic development of mechanics. Not surprisingly, his reasoning contains the same kinds of flaws that logicians now perceive in Euclid's work. Von Mises* has commented on this: "There are two new basic concepts that enter into the construction of Newtonian mechanics -- those of force and mass. Newton explains mass in his first definition as 'quantity of matter'. One notices immediately that this definition is completely empty and in no way helps us to gain an understanding of the phenomena of motion. All one has to do is reflect that it would be possible to substitute the words 'quantity of matter' for the word 'mass' where-ever it appears in a contemporary text. Newton's definition of force largely anticipates the content of the first two laws of motion: the force neither changes the location of a body (immediately), nor determines the velocity, but rather changes its velocity. Thus, force is first defined as something that changes the velocity, and then the law is stated that velocities are changed by forces. This manner of inference has been well put by Molière: 'The poppy seed is soporiferous; why? - because it has the power of soporificity.' But scoffing here is ill-advised, for Newton's Principia expresses one of the most far-reaching and original discoveries ever made in physics."

TIME AND KINEMATICS. The axiomatic method has had its most striking successes in geometry. Characteristically, axiomatic theories deal with undefined elements. For example, in the treatise on projective geometry by Veblen and Young, a point is undefined. A line is said to be an undefined set of points, except insofar as it is defined implicitly by certain postulates; e.g., two distinct points are on one and only one line; there are more than two points on a line, etc. A far-reaching hierarchy of theorems issues from such simple postulates.

Newton was aware that an axiomatic theory necessarily contains some undefined elements. In his Principia, he states that time, space, motion, and location do not require definition. The public has recognized vaguely, since the popularization of Einstein's theory, that our innate concept of time is wrong, or, at least, inappropriate for cosmology. However, this circumstance calls for no apologia, since, insofar as an axiomatic theory of Newtonian mechanics is concerned, the meaning of time is irrelevant. Consequently, the burden of defining and measuring time is transferred to philosophers and experimenters. For example, suppose that a position vector $\overline{r}$ is a function of an undefined parameter t; i.e., $\overline{r} = \overline{r}(t)$. This vector equation defines a curve C. Vectors $\overline{v}$

*Mathematical Postulates and Human Understanding, The World of Mathematics, vol. 3, edited by J. R. Newman, Simon & Schuster, New York, 1956.

and $\overline{a}$ are defined by $\overline{v} = d\overline{r}/dt$ and $\overline{a} = d\overline{v}/dt$. Then, by differential geometry, $\overline{v}$ is tangent to curve C, and $v = ds/dt$, where s is arc length on C. Furthermore,

$$\overline{a} = \frac{d\overline{v}}{dt} = \frac{d\overline{v}}{ds}\frac{ds}{dt} = v\frac{d\overline{v}}{ds} = v\frac{d}{ds}(v\hat{v}) = v^2\frac{d\hat{v}}{ds} + \overline{v}\frac{dv}{ds}$$

in which $\overline{v} = v\hat{v}$; i.e., $\hat{v}$ is the unit tangent vector of C. One of the formulas of Frenet in the differential geometry of curves is $d\hat{v}/ds = \hat{n}/R$, in which $\hat{n}$ is the principal unit normal vector of C, and $1/R$ is the curvature of C. Consequently,

$$\overline{a} = \hat{n}\frac{v^2}{R} + \hat{v}v\frac{dv}{ds} \qquad \text{or} \qquad \overline{a} = \hat{n}\frac{v^2}{R} + \hat{v}\frac{dv}{dt}$$

Accordingly, $\overline{a}$ lies in the osculating plane of curve C. The component of $\overline{a}$ on the principal normal is v^2/R and the component of $\overline{a}$ on the tangent to C is $dv/dt = d^2s/dt^2$. These are familiar results. The point is that the concept of time plays no role whatever in the deductions; t could be the x-coordinate, arc length on the curve, distance from the origin, or any other parameter.

Kinematics of a rigid body is essentially the theory of transformations of rectangular coordinates, in which the direction cosines of one set of axes with respect to the other are functions of a parameter t. If the two coordinate systems (x, y, z) and (ξ, η, ζ) have a common origin (which is no essential restriction, since translation is easily superimposed), and if the table of direction cosines is

	x	y	z
ξ	l_1	m_1	n_1
η	l_2	m_2	n_2
ζ	l_3	m_3	n_3

we obtain, by differentiating the equations of coordinate transformation, $\dot{x} = \dot{l}_1\xi + \dot{l}_2\eta + \dot{l}_3\zeta$, $\dot{y} = \ldots.$, $\dot{z} = \ldots.$, provided that the point under consideration has constant coordinates (ξ, η, ζ). If we eliminate (ξ, η, ζ) from these equations by means of the equations of coordinate transformation,

$$\xi = l_1 x + m_1 y + n_1 z, \ \ldots., \ \ldots.,$$

and simplify the resulting equations by means of the equations,

$$l_1 \dot{l}_1 + l_2 \dot{l}_2 + l_3 \dot{l}_3 = 0, \ldots, \ldots$$

$$l_1 \dot{m}_1 + l_2 \dot{m}_2 + l_3 \dot{m}_3 = - m_1 \dot{l}_1 - m_2 \dot{l}_2 - m_3 \dot{l}_3, \ldots, \ldots$$

which result by differentiation of algebraic identities among the direction cosines, we obtain

$$\dot{x} = z\,\omega_y - y\,\omega_z, \quad \dot{y} = x\,\omega_z - z\,\omega_x, \quad \dot{z} = y\,\omega_x - x\,\omega_y \qquad \text{(a)}$$

where, by definition,

$$\omega_x = m_1 \dot{n}_1 + m_2 \dot{n}_2 + m_3 \dot{n}_3$$

$$\omega_y = n_1 \dot{l}_1 + n_2 \dot{l}_2 + n_3 \dot{l}_3$$

$$\omega_z = l_1 \dot{m}_1 + l_2 \dot{m}_2 + l_3 \dot{m}_3 \qquad \text{(b)}$$

In modern engineering mechanics texts, Eq. (a) usually is derived in the vector notation $\overline{v} = \overline{\omega} x \overline{r}$ by reference to a vector diagram. However, that approach does not provide Eq. (b), which is needed to prove that $\overline{\omega}$ is a vector. If we introduce new initial coordinates (x', y', z') with constant direction cosines, given by

	x	y	z
x'	a_1	b_1	c_1
y'	a_2	b_2	c_2
z'	a_3	b_3	c_3

and if, by analogy to Eq. (a), we define $(\omega'_x, \omega'_y, \omega'_z)$ by

$$\dot{x}' = z'\,\omega'_y - y'\,\omega'_z, \quad \dot{y}' = x'\,\omega'_z - z'\,\omega'_x, \quad \dot{z}' = y'\,\omega'_x - x'\,\omega'_y \qquad \text{(c)}$$

the proof that angular velocity is a vector is equivalent to showing that it transforms like (dx, dy, dz); i.e.,

$$\omega'_x = a_1\,\omega_x + b_1\,\omega_y + c_1\,\omega_z, \ldots, \ldots \qquad \text{(d)}$$

Surprisingly, Eq. (d) does not follow from Eqs. (a), (c), and the equations of coordinate transformation, $x' = a_1 x + b_1 y + c_1 z, \ldots, \ldots$. These equations yield

$$\dot{x}' = a_1 \dot{x} + b_1 \dot{y} + c_1 \dot{z} = a_1 (z\,\omega_y - y\,\omega_z) + b_1 (x\,\omega_z - z\,\omega_x) \qquad \text{(e)}$$

$$+ c_1 (y\,\omega_x - x\,\omega_y) = (a_3 x + b_3 y + c_3 z)\,\omega'_y - (a_2 x + b_2 y + c_2 z)\,\omega'_z ,$$

$$\ldots\ldots\ldots, \quad \ldots\ldots\ldots$$

Equations (e) do not determine $(\omega'_x, \omega'_y, \omega'_z)$ uniquely because their determinant is zero. Their matrix is of rank 2. Because of identities among the direction cosines, the rank of the augmented matrix also is 2. One solution of Eq. (e) is Eq. (d). However, there are infinitely many solutions. We can let ω'_z be any function of (x, y, z) at pleasure, and solve Eqs. (e) for ω'_x and ω'_y. Consequently, something more than Eq. (a) and the equations of coordinate transformation from (x, y, z) to (x', y', z') is required to verify that angular velocity is a vector. The additional information that is needed is contained in Eq. (b).

The concepts of time and motion do not enter into the preceding analysis. In fact, the entire theory of kinematics of a rigid body may be regarded as geometry. Likewise, the philosophic interpretation of time is irrelevant in the kinematics of fluids. This is apparent if the displacement of a fluid is represented in the Lagrangian form, $\bar{r} = \bar{f}(\bar{r}_0, t)$, in which t is any parameter, and $\bar{f}(\bar{r}_0, 0) = \bar{r}_0$. The velocity field is defined by $\bar{v} = \partial\bar{r}/\partial t$, and the acceleration field by $\bar{a} = \partial\bar{v}/\partial t$. This characterization may be applied to any mechanical system, whether fluid or solid.

MASS AND THE LAWS OF DYNAMICS. It has been shown that time plays the role of an arbitrary scalar parameter in Newtonian kinematics. Mass is another scalar that may be defined incompletely in a postulational treatment of mechanics. Despite von Mises' critique of Newton's argument, the concept of mass as quantity of matter properly conveys the idea that mass is independent of gravity. On the basis of the atomic theory, quantity of matter may be defined as a measure of the numbers of protons and neutrons in a system.

The mass of a system is the sum of the masses of its parts. A mechanical system may be conceived as a set of particles, which generally is non-enumerable. In the language of mathematics, a configuration of the system is represented by a bounded Borel point set B. A subset of B has mass m. If we emulate the rigor of modern mathematics, we specify that m is a non-negative additive set function defined on all Borel subsets of B. The function m is invariant, in the sense that it is unchanged by a displacement of the system, i.e., m has the same value for all sets that are equivalent under the family of mappings $\bar{r} = \bar{f}(\bar{r}_0, t)$.

A velocity field $\overline{v}$ on the point set B is defined by $\overline{v} = \partial\overline{r}/\partial t$, and the momentum of system B is defined by

$$\overline{G} = \int_B \overline{v}\, dm ,$$

in which, for inclusion of point masses in the system and other generalities, the integral should be interpreted in the Lebesgue-Stieltjes sense. Also, the angular momentum of the system about the origin is defined by

$$\overline{H} = \int_B \overline{r} \times \overline{v}\, dm .$$

The fundamental equations of Newtonian dynamics are

$$\overline{F} = \frac{d\overline{G}}{dt}, \quad \overline{M} = \frac{d\overline{H}}{dt} \qquad \text{(f)}$$

These equations may be regarded as postulates. Since Newton's second law follows from $\overline{F} = d\overline{G}/dt$, the momentum principle is as broad as Newton's second law.

The practical significance of an axiomatic theory of mechanics lies in the fact that the variables m, t, $\overline{F}$, $\overline{M}$, etc. have prototypes in the real world that conform closely with the preceding equations and definitions. For certain physical systems (e.g., magnetohydrodynamic systems), the angular momentum principle may be invalid, but this circumstance does not vitiate the postulate $\overline{M} = d\overline{H}/dt$; it merely restricts applications of this equation. The equation $\overline{M} = d\overline{H}/dt$ is applicable only if the internal forces exert no resultant moment. The nature of interatomic forces often is adduced as evidence that this condition is satisfied. Symmetry of the stress tensor is a closely related condition.

If $\overline{F} = \overline{M} = 0$, and if the initial conditions are $\overline{G}(0) = \overline{H}(0) = 0$, Eqs. (f) show that $\overline{G}(t) = \overline{H}(t) = 0$ for $t > 0$. This is true for any bounded system, regardless of flexibility or fluidity, provided that the equation $\overline{M} = d\overline{H}/dt$ is applicable. If the system is rigid, the conditions $\overline{G} = \overline{H} = 0$ signifiy that there is no motion. Thus, principles of statics issue from the momentum principles.

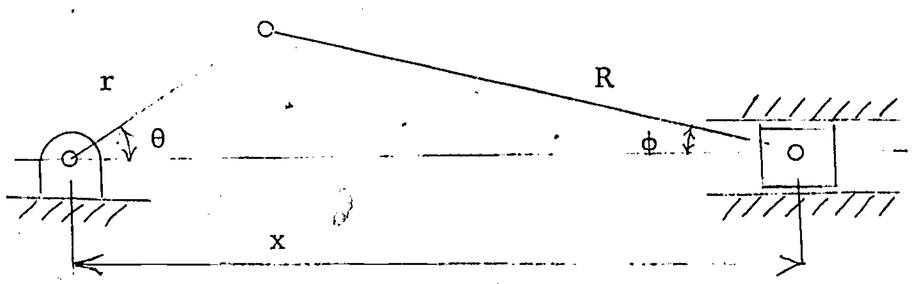
ON THE TEACHING OF KINEMATICS. A postulational development of Newtonian mechanics seemingly provides no important generalizations, such as those which have characterized geometry. Also, it would be too abstract for beginners. Insofar as possible, we should make education a continuous process. Continuity demands that we build on the knowledge that students already possess, and that we do not leap to new concepts and methods for which they are psychologically and educationally unprepared. Geometry is a prerequisite

for mechanics, but students often are muddled in mechanics because they are poorly prepared in Euclid's theorems, algebra, trigonometry, analytic geometry, and elementary calculus. The fact that kinematics is the branch of mechanics that causes beginners the most difficulty is evidence of this deficiency. Students get much-needed drill in mathematics if they derive linear and angular velocities and accelerations of parts of mechanisms by differentiating general geometric equations with respect to t. With the burgeoning applications of computers in kinematical design, this method has become important in practice. It is the approach which serves for the derivation of the equations of relative velocity and relative acceleration that have been used widely in analyses of mechanisms. It also is the phase of kinematics that is most important for dynamics. Graphical kinematics and the associated principles may be curtailed, since they have little bearing on dynamics. For example, for the slider-crank mechanism shown in the following figure:

$$x = r \cos\theta + R\cos\phi, \qquad \frac{\sin\theta}{R} = \frac{\sin\phi}{r}$$

Differentiating these equations with respect to t, and eliminating $\dot{\phi}$ and $\ddot{\phi}$, we get (with $\dot{\theta} = \omega$ = constant)

$$\dot{x} = -\frac{r\omega\sin(\theta+\phi)}{\cos\phi}, \qquad \ddot{x} = \frac{-r\omega^2}{\cos\phi}\left[\cos(\theta+\phi) + \frac{r\cos^2\theta}{R\cos^2\phi}\right]$$



Courses in dynamics usually begin with kinematics of a particle that moves on a straight axis. Accordingly, at the outset, we have the equations,

$$v = \frac{dx}{dt}, \qquad a = \frac{dv}{dt} = v\frac{dv}{dx} = \frac{d^2x}{dt^2}$$

The equation $a = v\, dv/dx$ sometimes creates the misconception that $a = 0$ whenever $v = 0$. However, it often happens that $dv/dx \to \infty$ as $v \to 0$. For example, for the slider-crank mechanism, let $\phi \to 0$, then $v \to 0$, $dv/dx \to \infty$, and $v\, dv/dx \to -r\omega^2$ $(\pm 1 + r/R)$. The last equation represents the accelerations of the slider at the ends of the stroke.

For a rotating body, there are relationships analogous to those for rectilinear motion of a particle; namely

$$\omega = \frac{d\theta}{dt}, \qquad a = \frac{d\omega}{dt} = \omega \frac{d\omega}{d\theta} = \frac{d^2\theta}{dt^2}$$

Consequently, it is natural to introduce the kinematics of rectinlinear motion and rotation about a fixed axis together. This approach expands the scope of problems (particularly, mechanism problems) that can be used for illustrations and exercises. Applications of the theory of rectilinear motion alone are quite limited, and the textbook problems in this area are unavoidably tri e or artificial.

A topic that usually is vague in the minds of students is relative motion. It is essential to emphasize that velocities and accelerations always are measured with respect to reference frames. Even for relative motion on a straight axis, the concept of reference frames is important, as Einstein's special theory of relativity shows. Understanding of relative motion will be enhanced if teachers designate reference frames explicitly. The difference $\overline{v}_2 - \overline{v}_1$ of the velocities of two particles customarily is called the velocity of particle 2 relative to particle 1, but, more precisely, it is the velocity of particle 2 relative to a reference frame that translates with particle 1. The same remark applies for the difference $\overline{a}_2 - \overline{a}_1$ of two accelerations. One fault of Gibbs' vector analysis is that it does not clearly display the fact that a vector can be specified only by means of three scalars which are associated with some reference frame. If, for example, a surveyor goes out in the field and measures a certain vector, he must record three numbers. The earth is his reference frame. It is easy for students to understand that, if (x, y, z) are rectangular coordinates of a particle, the velocity and the acceleration of the particle relative to that coordinate system are represented by $(\dot{x}, \dot{y}, \dot{z})$ and $(\ddot{x}, \ddot{y}, \ddot{z})$, regardless of the motion of the reference frame to which the axes (x, y, z) are attached. By making vectors appear absolute, and by renouncing the mathematical definition of a vector as a triplet of numbers that transforms like (dx, dy, dz), Gibbs' vector analysis sometimes tends to impede understanding. On the other hand, it appeals to the imagination, and it opens new mathematical vistas for thoughtful minds. Whether students of elementary mechanics generally have the necessary mathematical maturity to profit from symbolic vector analysis is debatable.

A serious detriment to the learning of elementary mechanics is the tendency of students to memorize special formulas. How often do they apply $v = at$ or $s = \frac{1}{2}at^2$ in cases for which the acceleration a is not constant? If special formulas did not stand out like captions in textbooks, students might be less inclined to memorize them. The fragmentation and cataloging of instructional material in textbooks tends to blur general principles, rather than to bring the mind to focus on them.

ON THE TEACHING OF DYNAMICS. The invariance of Newton's second law and of the principles of linear and angular momentum under Galilean transformations often is by-passed in elementary mechanics courses, although it is a feature of mechanics that is philosophically important and useful in practice. For instance, in studies of progressive waves, a reference frame that travels with the waves sometimes is preferable to a so-called fixed reference frame. Also, the earth is not exactly a Galilean reference frame, and the identification of astronomical reference frames for which Newton postulated his laws to be valid is not only essential for an understanding of mechanics, but it is fundamental in celestial mechanics, space navigation, meteorology, oceanography, and other fields.

Non-Galilean reference frames frequently are useful. For instance, I recently was asked about pressure cycles in a liquid carried in a hollow reciprocating piston that is filled with liquid. In such problems, the concept of inertial force is very helpful. Also, it is valuable in vibration theory, since, for example, the differential equations for a vibrating structure can be derived from the differential equations of statical equilibrium by introduction of inertial loads. This method is particularly convenient in the theories of vibration of beams, plates, and shells.

The work that a force $\overline{F}$ performs during a time interval (t_0, t_1) is defined as

$$W = \int_{t_0}^{t_1} \overline{F} \cdot \overline{v} \, dt , \tag{g}$$

in which $\overline{v}$ is the velocity of the particle on which the force acts. Since $\overline{v}$ depends on the choice of the reference frame, W also depends on the choice of the reference frame. Accordingly, work is a relative quantity. This observation is consistent with the law that the total work performed on a system equals the increase of kinetic energy, for kinetic energy also depends on the reference frame.

There is a subtlety in the definition of work if the force $\overline{F}$ does not act continually on the same particle. In this case Eq. (g) is valid, with the understanding that $\overline{v}$ is the velocity of the particle on which $\overline{F}$ acts, and not the velocity of the point of action of $\overline{F}$.

This distinction was emphasized by Osgood*. For example, if a rigid wheel rolls on a rigid track, the force $\overline{F}$ that the track exerts on the wheel may have a tangential component (e.g., if brakes are being applied). The velocity $\overline{v}$ of the particle of the wheel on which $\overline{F}$ acts is zero, since this particle lies at the instantaneous center. Therefore, force $\overline{F}$ performs no work on the wheel, despite the fact that the geometrical point of action of $\overline{F}$ moves along the track with the wheel. Similarly, if a grinding wheel acts on a fixed plate, the frictional force of the grinder performs no work on the plate since the plate does not move. However, the frictional force of the plate performs negative work on the grinder. The general definition of work deserves more emphasis, for there are many cases in which a force shifts from one particle of a body to another in a continuous way.

Unfortunately, the amount of important instructional material in mechanics usually is too great for the allotted class time. Consequently, teachers must weed out irrelevant and ancillary material. One topic that may be dropped is the coefficient of restitution. This concept is an empiricism, and, unlike the coefficient of friction, it has little importance. Serious studies of impact lead to considerations of elastic or inelastic deformation and wave motion which are beyond the scope of elementary mechanics. If the coefficient of restitution is omitted, there is little need for the concept of impulse. Whether the momentum principle is written as $\overline{F} = d\overline{G}/dt$ or $\Delta\overline{G} = \int_{t_0}^{t_1} \overline{F}\, dt$ is mathematically optional, but the first form is simpler, both in form and conception. In fluid mechanics, the momentum principle is best expressed with reference to convection of momentum through a control surface; i.e., the resultant external force that acts on the fluid in a given spatial region R equals the net rate at which momentum is convected out of the region R, plus the time rate of increase of momentum of fluid in the region R. The principle of angular momentum in fluid mechanics may be stated similarly; we merely replace the words "force" and "momentum" by the phrases "moment of force" and "moment of momentum." These principles have nothing to do with special properties of fluids, and they properly belong in a general mechanics course.

UNITS OF MEASUREMENT. The want of a philosophical attitude is apparent, not only among students, but also among practicing engineers, when they confuse the concepts of weight and mass. This inability to discriminate between physical concepts that are entirely different is aggravated by the ill-chosen units called "pounds force" and "pounds mass", or "kilograms force" and "kilograms mass." Conceivably, we might adopt a unit of time

*W. F. Osgood, Mechanics, Chap. 7, Art. 7, The Macmillan Co., New York, 1937.

called the meter (as musicians do). Then, since the meter also is a unit of length, we would have to distinguish between "meters length" and "meters time." The ambiguous use of the pound or the kilogram as a unit of mass and a unit of force slipped innocuously into thermodynamics because there is no dynamics in classical thermodynamics, except the concept of work; the equation $F = ma$ plays no role. In modern gas dynamics, where the equation $F = ma$ is involved, the ambiguity has been retained by means of the modified Newtonian equation $F = ma/g_c$. This practice further muddles the concepts of weight and mass; students are unable to decide whether gravity affects a phenomenon or not.

Hydraulicists are consistent in their units. In the United States, they have used the slug and the pound as units of mass and force, respectively. Bernoulli's equation, $p/w + v^2/2g + Z = C$, requires that w be interpreted as specific weight. Although this equation is correct, it is misleading because g is in the wrong place. For instance, flow in a horizontal conduit is unaffected by gravity, but g appears in Bernoulli's equation. Lord Rayleigh called attention to this oddity in many engineering formulas. He said, "When the question under consideration depends essentially upon gravity, the symbol g makes no appearance, but, when gravity does not enter into the question at all, g obtrudes itself conspicuously." The universal hydraulic practice of defining heads as lengths tends to promulgate this deception. It is true that head represents energy per unit weight of fluid, but the predominance of the concept of weight is itself misleading. In the equation of gas dynamics, $p/\rho + \frac{1}{2}v^2 + gz + u = C$, each term represents energy per unit mass. This is a more rational way to express specific energy. The misplaced g pervades all applied mechanics, for mass m commonly is represented as W/g. This practice gives the false implication that W is invariant, and that m depends on g -- a misconception that many students carry away from college into their work. It is more in accord with the nature of things to write $W = mg$, and to specify masses of objects, rather than weights. Then, if gravity has no effect on a phenomenon, g does not appear in the related equations.

The confusion is not restricted to the English system, for European engineers interpret the kilogram as a unit of force, as we see from the practice of expressing stresses in the unit kg/mm^2. The kilogram also is used as a unit of mass, but, when Newton's law, $F = ma$, is applied, the unit of mass has been taken to be the $kg\ sec^2/m$. The Système Internationale that is now accepted by nearly all engineering societies is a supreme effort to end the ambiguity, but there is a grave danger that practitioners will persist in using the kilogram as a unit of force. Concerted efforts by textbook writers and research workers will be needed to imbue the next generation of engineers with the idea that the kilogram is a unit of mass, and that it never is a unit of force.

CONCLUDING REMARKS. Rigor will be enhanced if numerical problems are de-emphasized in favor of more weight on concepts, derivations, and interpretations of principles and equations. If algebraic symbols are specified for lengths, masses, forces, velocities, etc. in exercises, rather than numerical values, the solution to each problem becomes, in itself, a minor derivation. Such general solutions to rather special problems often are used in practice.

In a broad sense, rigor means straight thinking or good reasoning. If we accept this definition, the need for rigor in the teaching of mechanics is a truism. Perhaps the greatest detriment to straight thinking in mechanics, and parenthetically in other branches of physics, is the want of a philosophical attitude. Habits of contemplation which are essential for understanding of mathematics and physics are not cultivated in engineering colleges more than in the past. Possibly teachers can do little to develop them, but educational experiments in this direction are rare. Students in engineering seldom are required to master deductive proofs, yet real understanding of fundamentals comes only from such mastery. Nothing is more practical than a good theory, and the key to proper applications of the theory is comprehension of the development of the theory.